SAFE HANDS & IIT-ian's PACE

LEAP TEST# 11 (JEE) ANS KEY Dt. 29-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	D	31	B	61	B
2	B	32	B	62	C
3	C	33	B	63	B
4	A	34	D	64	D
5	A	35	B	65	A
6	D	36	A	66	A
7	C	37	A	67	B
8	C	38	C	68	C
9	D	39	A	69	A
10	A	40	A	70	C
11	A	41	A	71	D
12	C	42	A	72	A
13	C	43	B	73	D
14	C	44	D	74	B
15	A	45	D	75	A
16	C	46	A	76	A
17	D	47	B	77	A
18	B	48	D	78	D
19	A	49	C	79	C
20	A	50	B	80	C
21	0.49	51	6	81	7
22	87.91	52	4	82	4
23	2	53	2	83	64
24	66.67	54	6	84	30
25	2.82	55	4	85	3

See Physics & Maths solutions on next page.....

SAFE HANDS & PACE

LT 11 (JEE) Physics Solutions

: ANSWER KEY :

1)	d	2)	b	3)	c	4)	a	21)	0.49	22)	87.91	23)	2
5)	a	6)	d	7)	c	8)	c	24)	66.67				
9)	d	10)	a	11)	a	12)	c	25)	2.82				
13)	c	14)	c	15)	a	16)	c						
17)	d	18)	b	19)	a	20)	a						

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(d)**

$$f=\frac{P}{2L}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow p_1\sqrt{T_1}=p_2\sqrt{T_2}$$

$$\Rightarrow 6\sqrt{36}=4\sqrt{T_2}\Rightarrow T_2=81\text{ N}$$

2 **(b)**

$$v=n\lambda$$

$$=2n(l_1-l_2)=2f\times 1=2f\text{ m/s}$$

3 **(c)**

Beat frequency $=f_1-f_2=\frac{v}{2l}-\frac{v}{2(1+x)}$

$$=\frac{v}{2l}\left[1-\left(1+\frac{x}{l}\right)^{-1}\right]$$

$$=\frac{v}{2l}\left[1-1+\frac{x}{l}\right]$$

$$=\frac{vx}{2l^2}$$

4 **(a)**

According to Hooke's law, $F_g\propto x$ [Restoring force $F_g=T$, tension of spring]

Velocity of sound by a stretched string

$$v=\sqrt{\frac{T}{m}}$$

Where m is the mass per unit length

$$\therefore\ \frac{v}{v'}=\sqrt{\frac{T}{T'}}\Rightarrow v'=v\sqrt{\frac{T'}{T}}=v\sqrt{\frac{1.5x}{x}}=1.22v$$

5 **(a)**

$\lambda=2l=3$ m

Equation of standing wave

(As $x=0$ is taken as a node)

$y=2A\sin kx\cos\omega t$,

Given $2A=4$ mm

To find value of x for which amplitude is 2 mm, we have 2 mm = (4 mm) sin kx

$$\frac{2\pi}{\lambda}x=\frac{\pi}{6}\Rightarrow x_1=\frac{1}{4}\text{m}$$

$$\frac{2\pi}{\lambda}x=\frac{\pi}{2}+\frac{\pi}{3}\Rightarrow x_2=1.25\text{ m}$$

$$x_2-x_1=1\text{ m}$$

6 **(d)**

$$\frac{n_1}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi r^2\rho}}$$

$$=\frac{n_2}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi(4r^2)\rho}}$$

$$\Rightarrow\frac{n_1}{n_2}=\frac{1}{2}$$

7 **(c)**

Given $v_c=v_o$(both first overtone)

Or

$$3\left(\frac{v_c}{4L}\right)=2\left(\frac{v_o}{2l_o}\right)$$

$$\therefore\ l_o=\frac{4}{3}\left(\frac{v_o}{v_c}\right)L=\frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}L$$

$$\left(as\ v\propto\frac{1}{\sqrt{\rho}}\right)$$

Therefore correct option is (c).

8 **(c)**

Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence, options (a) and (b) both are wrong. To obtain a stationery wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either both of the waves should be longitudinal or both of them should be transverse. Hence, option (c) is correct

9 **(d)**

$$I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$$

$$\Rightarrow 7I=I+9I+2\sqrt{I9I}\cos\phi$$

$$\cos\phi=-1/2\ \text{ or }\ \phi=120°$$

10 **(a)**

Slope at any point on the string in wave motion represents the ratio of particle speed to wave speed

Therefore, slope $B<$ slope A

Hence $R_A>R_B$

11 **(a)**

$$l_1+\varepsilon=\frac{v}{4f_0}$$

$$l_2+\varepsilon=\frac{3v}{4f_0}$$

$$l_3+\varepsilon=\frac{5v}{4f_0}$$

On solving, we get $l_3 = 2l_2 - l_1$

12 **(c)**

For destructive interference, path difference has to be equal to an odd integral multiple of $\lambda/2$

13 **(c)**

$3 \times \frac{v}{4l_c} = 2 \times \frac{v}{2l_0}$ or $\frac{l_c}{l_0} = \frac{3}{4}$

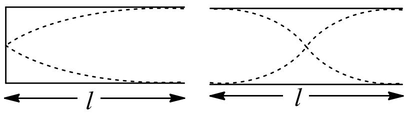
14 **(c)**

$f = \frac{v}{4l} = \frac{320}{4}$ Hz $= 80$ Hz

Since even harmonic cannot be present therefore 320 Hz ($= 4 \times 80$) is ruled out

15 **(a)**

$\lambda/4 = l$ (Fundamental mode), $\lambda = 4l, c = v\lambda$



$\therefore\ v = \frac{c}{\lambda} = \frac{c}{4l} = 512$ Hz

Given, $\lambda'/2 = l$

Fundamental mode,

$\therefore\ \lambda' = 2l$ but $c = v'\lambda'$

$\therefore\ v' = \frac{c}{\lambda'} = \frac{c}{2l} = 2\left(\frac{c}{4l}\right)$

$= 2 \times 512 = 1024$ Hz

16 **(c)**

Beats $= \frac{V}{4l} - \frac{V}{4(l+\Delta l)} = \frac{V}{4}\left[\frac{\Delta l}{l(l+\Delta l)}\right]$

$= \frac{V\Delta l}{4l^2}$ $(\because \Delta l \ll l)$

17 **(d)**

Given that the frequency of wave produced if the string is $1/n$

$\therefore\ \frac{1}{n} = \frac{1}{2\pi}\sqrt{\frac{T}{m}}$

Now $T' = 2T$

Therefore, new frequency is

$f = \frac{1}{2\pi}\sqrt{\frac{2T}{m}} = \sqrt{2} \times \frac{1}{n}$

Therefore, the number of waves produced per second is

$\frac{1}{f} = \frac{n}{\sqrt{2}}$

18 **(b)**

Fundamental frequency of a COP is given by $f_1 = v/4l$

Length l of the column will first descrease and then become constant (when rate or inflow=rate of outflow). Therefore f_0 will first increases and then become constant

Matrix Match Type

19 **(a)**

Wavelength of wave in medium changes when there is relative motion between medium and source. Frequency observed by observer is different from source frequency only if there is relative motion between observer and source. Speed of sound w.r.t. medium will not change until temperature of medium changes

20 **(a)**

Power $\propto f^2A^2$

Integer Answer Type

21 **(0.49)**

Given, audio output = 3W

Intensity, I = 120 dB

$I_0 = 10^{-12}\ \mathrm{W/m^2}$

From loudness relation,

$\Rightarrow \mathrm{dB} = 10 \log \frac{I}{I_0}$

$\Rightarrow 120 = 10 \log\left(\frac{I}{I_0}\right)$

$\Rightarrow I = 1\ \mathrm{W/m^2}$

Final intensity $= \frac{P_{out}}{4\pi r^2}$

$\Rightarrow I = \frac{P_{out}}{4\pi r^2}$

$\Rightarrow r = \sqrt{\frac{3}{4\pi(1)}} = \frac{0.98}{2} = 0.49$ m

22 **(87.91)**

$v = \sqrt{\frac{rRT}{M}}$ ... (i)and

$f = \frac{v}{\lambda}$ ... (ii)

From (i) and (ii)

$\Rightarrow f \propto \sqrt{T}$ (iii)

Let the frequency of tuning fork be f.

Thus, frequency of air column at 16°C (289 K) $= f + 4$

And frequency of air column at 10°C (283 K) $= f + 3$

From (iii)

$\Rightarrow \frac{f+4}{f+3} = \sqrt{\frac{289}{283}} = \frac{17}{\sqrt{283}} = \frac{17}{16.82}$

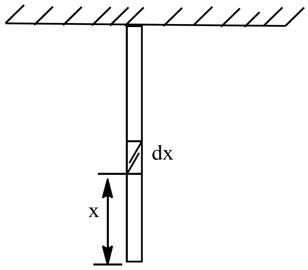
$\Rightarrow \frac{f+4}{f+3} = 1.011$

$\Rightarrow f + 4 = 1.011\,f + 3 \times 1.011$

$\Rightarrow (1.011 - 1)f = 4 - 3.033$

$\Rightarrow f = \frac{0.967}{0.011} = 87.91$ Hz

23 **(2)**



Caution:

1. Tension will increase as elevation x increases.

2. Wave speed increases with height.

Now, $\mu = \frac{M}{L}$

Tension in the rope at each point $'x' = \mu x g$

$$\Delta v = \sqrt{\frac{T}{\mu}} = \sqrt{gx}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{gx}$$

$$\Rightarrow dt = \frac{dx}{\sqrt{gx}}$$

Integrating both sides,

$$\Delta t = \frac{1}{\sqrt{gx}}\int_0^L \frac{dx}{\sqrt{x}} = \frac{\left[2\sqrt{x}\right]_0^L}{\sqrt{g}} = 2\sqrt{\frac{L}{g}} = 2\sqrt{\frac{10}{9.8}}$$

$$\therefore \Delta t = 2\left(\frac{\sqrt{50}}{7}\right) = 2\left(\frac{5\sqrt{2}}{7}\right)$$

$$= 2 \times 0.71 \times 1.41$$

$$= 2 \times 1.00 \text{ s}$$

$$\therefore \Delta t = 2 \text{ s}$$

24 **(66.67)**

$y = 0.07\cos(250t - 15x)$

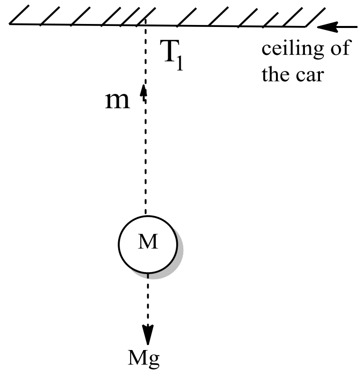
$v = \frac{\omega}{k} = \frac{250}{15} = \frac{50}{3}$ m/s

Also $v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow T = v^2\mu = \left(\frac{50}{3}\right)(4) = \frac{200}{3} = 66.67$ N

25 **(2.82)**

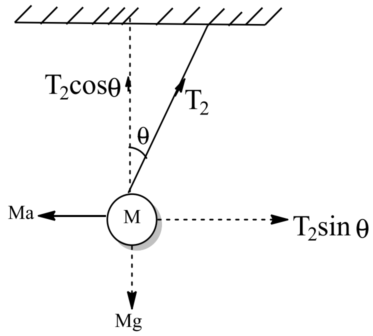
Case I:



From diagram,

$$\therefore v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{mg}{\mu}} \quad \ldots \text{(i)}$$

Case II:



From diagram,

$T_2 \cos\theta = mg, T_2 \sin\theta = ma$

$\therefore {T_2}^2(\sin^2\theta + \cos^2\theta)$

$T_2 = m(g^2 + a^2)^{1/2}$

$$v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{m\sqrt{a^2 + g^2}}{\mu}} \quad \ldots \text{(ii)}$$

From (i) and (ii),

$$\frac{v_1}{v_2} = \sqrt{\frac{g}{(g^2 + a^2)^{1/2}}}$$

$$\Rightarrow \left(\frac{v_2}{v_1}\right) = \left(\frac{g^2 + a^2}{g^2}\right)^{\frac{1}{4}}$$

Taking binomial approximation, we get

$$\frac{v_2}{v_1} = 1 + \frac{1}{4}\frac{a_2}{g_2} \quad \ldots [\because (1 + x)^n \approx 1 + nx]$$

$$\Rightarrow \frac{51}{50} = 1 + \frac{1}{4}\frac{a_2}{g_2}$$

$$\Rightarrow \frac{a_2}{g_2} = \frac{8}{100}$$

$$\Rightarrow a = \frac{2\sqrt{2}g}{10}$$

$$\Rightarrow a = 2.82 \text{ ms}^{-2}$$

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LT 11 JEE Mathematics Solutions

: ANSWER KEY :

61)	b	62)	c	63)	b	64)	d	81)	7	82)	4	83)	64
65)	a	66)	a	67)	b	68)	c	84)	30				
69)	a	70)	c	71)	d	72)	a	85)	3				
73)	d	74)	b	75)	a	76)	a						
77)	a	78)	d	79)	c	80)	c						

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(b)**

$$\int_0^1 \cot^{-1}(1-x+x^2)dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-x)dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}[1-(1-x)]\, dx$$

$$= 2\int_0^1 \tan^{-1} x dx \;\Rightarrow\; \lambda = 2$$

62 **(c)**

Put $x - 0.4 = t \Rightarrow \int_{0.6}^{3.6}\{t\}dt = \int_{0.6}^{0.6+3}\{t\}dt$

$$= 3\int_0^1 (t-[t])\, dt = 3\left(\frac{t^2}{2}\right)_0^1 = \frac{3}{2} = 1.5$$

63 **(b)**

$I = \int_{-3}^{3} x^8\{x^{11}\}dx(1)$

Replacing x by $-x$, we have $I = \int_{-3}^{3} x^8\,\{-x^{11}\}\;dx$ (2)

Adding equations (1) and (2), we get

$$2I = \int_{-3}^{3} x^8(\{x^{11}\} + \{-x^{11}\})dx$$

$$= 2\int_0^3 x^8 dx = 2\left(\frac{x^9}{9}\right)_0^3 = 2.3^7$$

$\Rightarrow\; I = 3^7$ [as$\{x\} + \{-x\} = 1$ for x is not an integer]

64 **(d)**

$\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + \; a$ (1)

For $x = 1, \int_0^1 f(t)dt = 0 + \frac{1}{8} + \frac{1}{3} + a = \frac{11}{24} + \; a$

Differentiating both sides of equation (1) w.r.t. x we get,

$f(x) = 0 - x^2 f(x) + 2x^{15} + 2x^5$

$$\Rightarrow\; f(x) = \frac{2(x^{15} + x^5)}{1+x^2}$$

$$\Rightarrow\; 2\int_0^1 \frac{x^{15} + x^5}{1+x^2}dx = \frac{11}{24} + a$$

$$\Rightarrow 2\int_0^1 (x^{13} - x^{11} + x^9 - x^7 + x^5)dx = \frac{11}{24} + a$$

$$\Rightarrow 2\left(\frac{1}{14} - \frac{1}{12} + \frac{1}{10} - \frac{1}{8} + \frac{1}{6}\right) = \frac{11}{24} + a$$

$$\Rightarrow\; a = -\frac{167}{840}$$

65 **(a)**

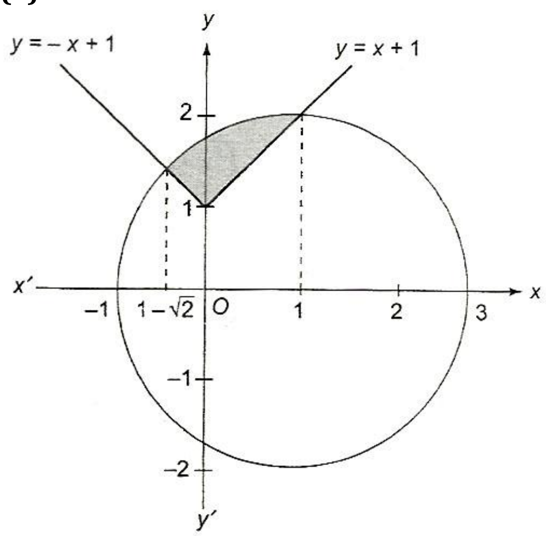
$I = \int_0^{\pi/2} \frac{\sin 2x}{x+1}dx$. Put $x = y/2$

$$\Rightarrow\; I = \int_0^{\pi} \frac{\sin y}{y+2}dy$$

$= \left(\frac{-\cos y}{y+2}\right)_0^{\pi} - \int_0^{\pi}\frac{\cos y}{(y+2)^2}dy$ (integrating by parts)

$$\Rightarrow\; I = \frac{1}{\pi+2} + \frac{1}{2} - A$$

66 **(a)**



$x^2 + y^2 - 2x - 3 = 0$

$\Rightarrow (x-1)^2 + y^2 = 4$

$$A = \int_{1-\sqrt{2}}^{0} (\sqrt{4-(x-1)^2} - (-x+1)dx$$

$$+ \int_0^1 \left(\sqrt{4-(x-2)^2} - (x+1)\right)dx$$

$$= \frac{x-1}{2}\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\frac{x-1}{2} + \frac{x^2}{2} - x\Big|_{1-\sqrt{2}}^{0}$$

$$+\frac{x-1}{2}\sqrt{4-(x-1)^2}+\frac{4}{2}\sin^{-2}\frac{x-1}{2}-\frac{x^2}{2}-x\Big|_0^1$$

$$=\left(-\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)-\left(\frac{-\sqrt{2}}{2}\sqrt{2}-\frac{\pi}{2}+\frac{3-2\sqrt{2}}{2}-1\right.$$
$$\left.+\sqrt{2}\right)+\left(-\frac{1}{2}-1\right)-\left(-\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)$$

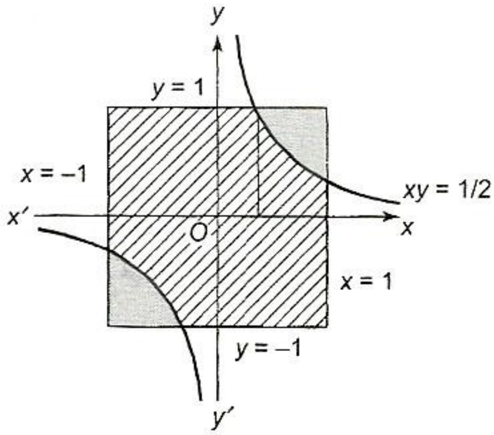
$$=-\left(-1-\frac{\pi}{2}+\frac{3}{2}-\sqrt{2}-1+\sqrt{2}\right)-\frac{3}{2}$$

$$=\frac{\pi}{2}-1 \text{ sq. units}$$

67 **(b)**

$\max(|x|,|y|)\le 1 \Rightarrow |x|\le 1$, and $|y|\le 1$

Which represent square bounded by $x=\pm 1$ and $y=\pm 1$



Required area is lined area

Now, shaded area is

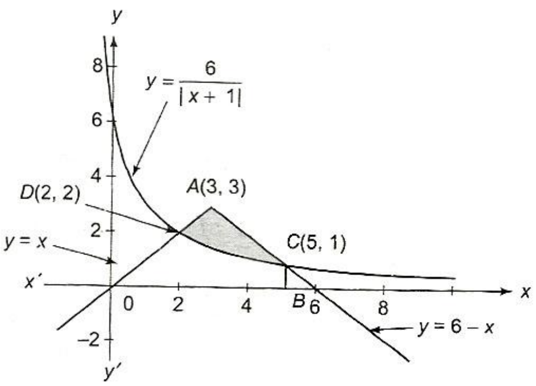
$$2\int_{1/2}^{1}\left(1-\frac{1}{2x}\right)dx=2\left(x-\frac{1}{2}\text{ In }x\right)_{1/2}^{1}$$

$$=2\left[(1-0)-\left(\frac{1}{2}-\frac{1}{2}\text{ In}\frac{1}{2}\right)\right]$$

$=1-\text{In }2$ sq. units

$\Rightarrow$ Horizontal lined area $=4-(1-\text{In }2)=3+\text{In }2$ sq. units

68 **(c)**



First consider $y=3-|3-x|$

For $x<3; y=3-(3-x)=x$

For $x\ge 3; y=3-(x-3)=6-x$

Consider $y=\frac{6}{|x+1|}$

For $x<-1; y=\frac{6}{-1-x}$

$\Rightarrow (1+x)y=-6$

For $x>-1; y=\frac{6}{x+1}$

Required area

$$=\left[\int_2^3\left(x-\frac{6}{x+1}\right)dx+\int_3^5\left((6-x)-\frac{6}{x+1}\right)dx\right]$$

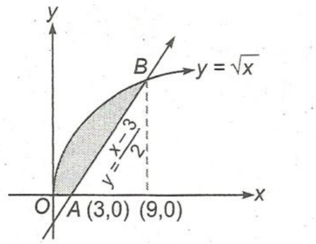
$$=\left[\left(\frac{x^2}{2}\right)_2^3+\left(6x-\frac{x^2}{2}\right)_3^5-(6\log(x+1))_2^5\right]$$

$$=\left[\frac{5}{2}+4-6\log 2\right]=\frac{13}{2}-6\text{ In }2 \text{ sq. units}$$

69 **(a)**

$$\text{Required area } OABO=\int_0^9\sqrt{x}\,dx-\int_3^9\left(\frac{x-3}{2}\right)dx$$

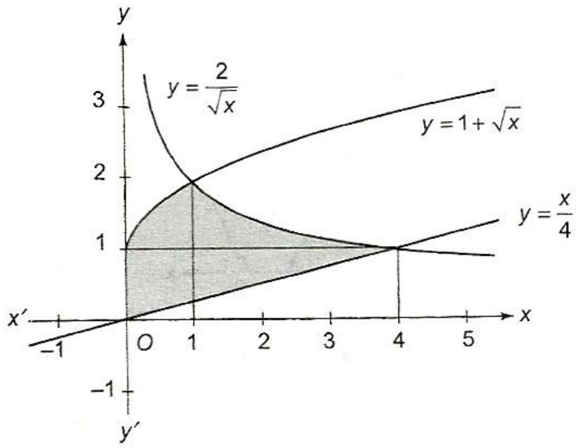
$$=\left(\frac{x^{3/2}}{3/2}\right)_0^9-\frac{1}{2}\left(\frac{x^2}{2}-3x\right)_3^9$$



$$=\left(\frac{2}{3}\cdot 27\right)-\frac{1}{2}\left\{\left(\frac{81}{2}-27\right)-\left(\frac{9}{2}-9\right)\right\}$$

$=9$ sq units

70 **(c)**



$$A_1=\int_0^1\left(1+\sqrt{x}-\frac{x}{4}\right)dx$$

$$=\left[x+\frac{2x^{3/2}}{3}-\frac{x^2}{8}\right]_0^1=1+\frac{2}{3}-\frac{1}{8}=\frac{37}{24}$$

$$A_2=\int_1^4\left(\frac{2}{\sqrt{x}}-\frac{x}{4}\right)dx$$

$$=\left[4\sqrt{x}-\frac{x^2}{8}\right]_1^4$$

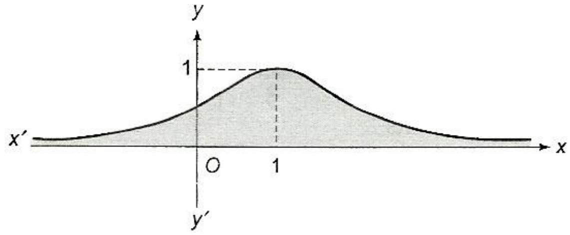
$$=\left[8-2-4+\frac{1}{8}\right]=\frac{17}{8}$$

$\Rightarrow A = A_1 + A_2 = \frac{88}{24} = \frac{11}{3}$ sq. units

71 **(d)**

$y = \frac{1}{(x-1)^2 + 1}$

y is maximum when $(x-1)^2 = 0$. Also, graph is symmetrical about line $x = 1$



Area $= 2\int_1^{\infty} \frac{1}{(x-1)^2+1} dx = 2[\tan^{-1}(x-1)]_1^{\infty} = \pi$ sq. units

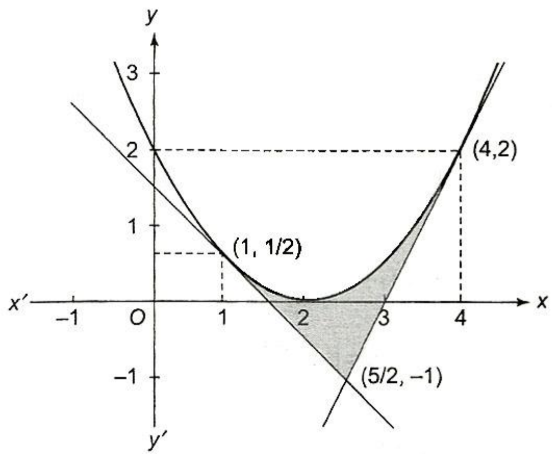
72 **(a)**

$y = \frac{x^2}{2} - 2x + 2 = \frac{(x-2)^2}{2}$,

$\frac{dy}{dx} = x - 2, \left(\frac{dy}{dx}\right)_{x=1} = -1, \left(\frac{dy}{dx}\right)_{x=4} = 2$

$\Rightarrow$ Tangent at $(1, 1/2)$ is $y - 1/2 = -1(x-1)$ or $2x + 2y - 3 = 0$

Tangent at $(4, 2)$ is $y - 2 = 2(x-4)$ or $2x - y - 6 = 0$



Hence, $A = \int_1^{5/2} \left(\frac{x^2}{2} - 2x + 2 - \frac{3-2x}{2}\right) dx + \int_{5/2}^{4} \left(\frac{x^2}{2} - 2x + 2 - (2x-6)\right) dx$

$$= \int_1^4 \left(\frac{x^2}{2} - 2x + 2\right) dx - \int_1^{5/2} \left(\frac{3-2x}{2}\right) dx - \int_{5/2}^{4} (2x-6) dx$$

$$= \left(\frac{x^3}{6} - x^2 + 2x\right)_1^4 - \frac{1}{2}(3x - x^2)_1^{5/2} - (x^2 - 6x)_{5/2}^4$$

$$= \left(\frac{63}{6} - 15 + 6\right) - \frac{1}{2}\left(3 \times \frac{3}{2} - \left(\frac{25}{4} - 1\right)\right) - \left(\left(16 - \frac{25}{4}\right) - 6\left(4 - \frac{5}{2}\right)\right)$$

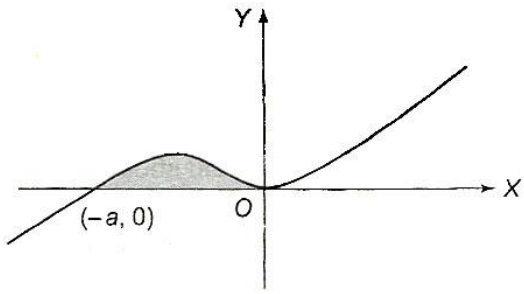
$$= \frac{3}{2} - \frac{1}{2}\left(\frac{9}{2} - \frac{21}{4}\right) - \left(\frac{39}{4} - 6\left(\frac{3}{2}\right)\right)$$

$$= \frac{9}{8} \text{ sq. units}$$

73 **(d)**

The curve is $y = \frac{x^2(x+a)}{a^2}$, which is a cubic polynomial

Since $\frac{x^2(x+a)}{a^2} = 0$ has repeated root $x = 0$, it touches x-axis at $(0, 0)$ and intersects at $(-a, 0)$



Required area $= \int_{-a}^{0} y dx = \int_{-a}^{0} \left[\frac{x^2(x+a)}{a^2}\right] dx = a^2/12$ sq. units

74 **(b)**

$xy^2 = a^2(a - x)$

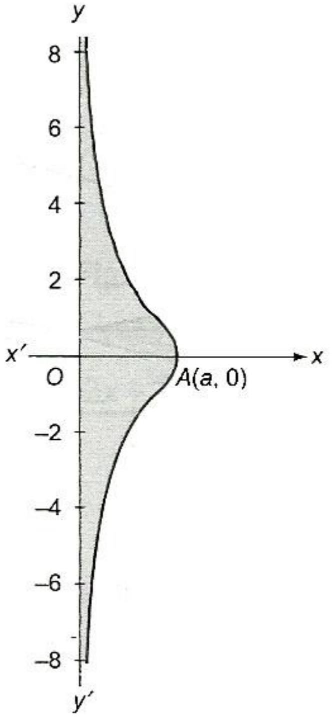
$\Rightarrow x = \frac{a^3}{y^2 + a^2}$

The given curve is symmetrical about x-axis, and meets it at $(a, 0)$

The line $x = 0$, i.e., y-axis is an asymptote (tangent at infinity)

Area $= \int_0^{\infty} x\, dy = 2\int_0^{\infty} \frac{a^3}{y^2+a^2} dx$

$$= 2a^3 \frac{1}{a}\left[\tan^{-1}\frac{y}{a}\right]_0^{\infty} = 2a^2 \frac{\pi}{2} = \pi a^2 \text{ sq. units}$$



75 **(a)**

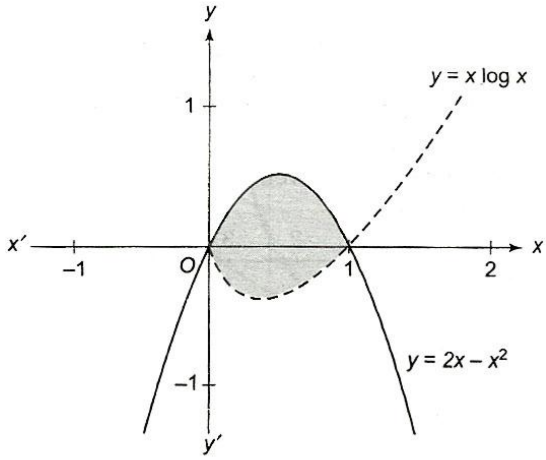
Curve tracing: $y = x\log_e x$

Clearly, $x > 0$

For $0 < x < 1, x\log_e x < 0$, and for $x > 1, x\log_e x > 0$

Also $x\log_e x = 0 \Rightarrow x = 1$

Further, $\frac{dy}{dx} = 0 \Rightarrow 1 + \log_e x = 0 \Rightarrow x = 1/e$, which is a point of minima



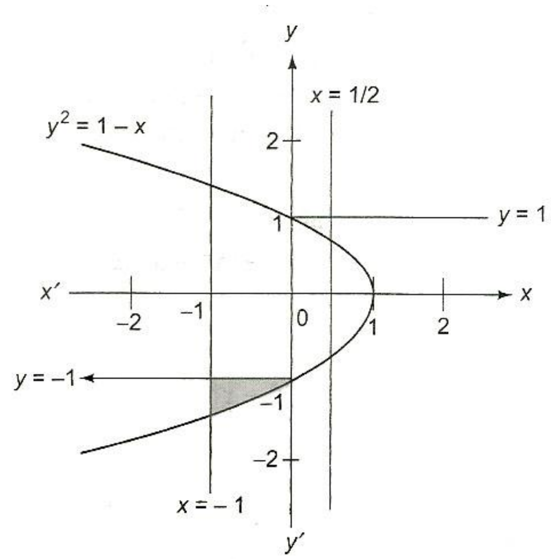
Required area

$$= \int_0^1 (2x - 2x^2)dx - \int_0^1 x\log x\, dx$$

$$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1$$

$$= \left(1 - \frac{2}{3}\right) - \left[0 - \frac{1}{4} - \frac{1}{2}\lim_{x\to 0} x^2\log x\right] = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

76 **(a)**



From figure

$$A = \int_{-1}^{0} \left(-1 - \left(-1\sqrt{1-x}\right)\right) dx + \int_0^{1/2} (1 - \sqrt{1-x})dx$$

$$= \left[-x - \frac{(1-x)^{3/2}}{3/2}\right]_{-1}^{0} + \left[x + \frac{(1-x)^{3/2}}{3/2}\right]_0^{1/2}$$

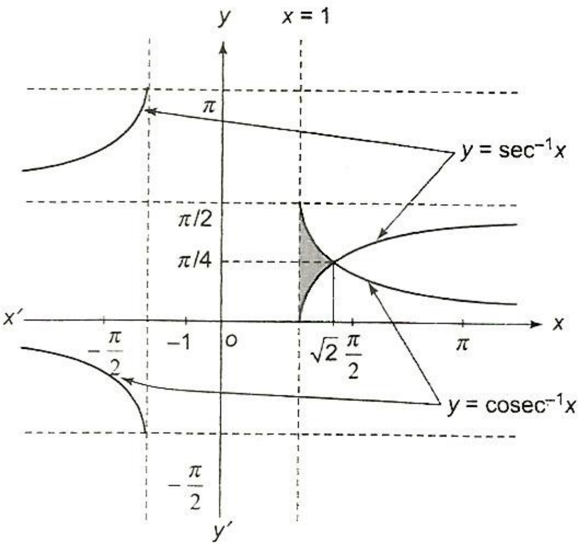
$$= \left[-\frac{2}{3} - \left(1 - \frac{2\times 2^{3/2}}{3}\right)\right] + \left[\frac{1}{2} + \frac{2}{3\times 2^{3/2}} - \frac{2}{3}\right]$$

$$= \frac{2}{3\times 2^{3/2}} + \frac{2\times 2^{3/2}}{3} - \frac{4}{3} - \frac{1}{2}$$

$$= \frac{3}{\sqrt{2}} - \frac{4}{3} - \frac{1}{2}$$

$$= \frac{3}{\sqrt{2}} - \frac{11}{6} \text{ sq. units}$$

77 **(a)**



Integrating along x-axis, we get

$$A = \int_1^{\sqrt{2}} (\text{cosec}^{-1}x - \sec^{-1}x)dx$$

Integrating along y-axis, we get

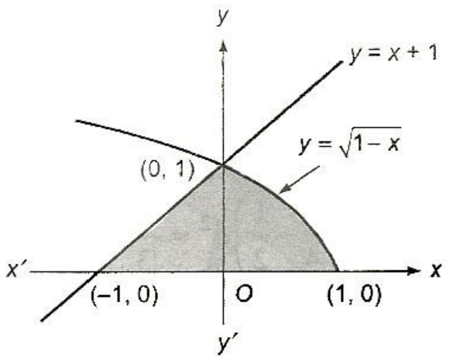
$$A = 2\int_0^{\pi/4} (\sec y - 1)dy$$

$= 2[\log|\sec y + \tan y| - y]_0^{\pi/4}$

$= 2\left[\log\left|\sqrt{2}+1\right| - \frac{\pi}{4}\right]$

$= \log\left(3+2\sqrt{2}\right) - \frac{\pi}{2}$ sq. units

78 **(d)**



Required area = shaded region

$= \int_0^1 (x_2 - x_1)dy$ (integrating along y-axis)

$= \int_0^1 [(1-y^2) - (y-1)]dy$

$= \frac{7}{6}$ sq. units

79 **(c)**

$a^2x^2 + ax + 1$ is clearly positive for all real values of x. Area under consideration

$A = \int_0^1 (a^2x^2 + ax + 1)\,dx$

$= \frac{a^2}{3} + \frac{a}{2} + 1$

$= \frac{1}{6}(2a^2 + 3a + 6)$

$= \frac{1}{6}\left(2\left(a^2 + \frac{3}{2}a + \frac{9}{16}\right) + 6 - \frac{18}{16}\right)$

$= \frac{1}{6}\left(2\left(a+\frac{3}{4}\right)^2 + \frac{39}{8}\right)$, which is clearly minimum for $a = -\frac{3}{4}$

80 **(c)**

$\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$

Differentiating both sides w.r.t. β, we get

$\therefore f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

$\Rightarrow f'(\beta) = -\beta\sin\beta + \cos\beta + \cos\beta - \frac{\pi}{4}\cos\beta$

$\Rightarrow f'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

Integer Answer Type

81 **(7)**

$F(x) = \int_0^x f(t)\,dt$

Differentiating with respect to x, we get

$F'(x) = f(x)$ ... (i)

$F(x^2) = x^2(1+x)$

$\Longrightarrow F(x^2) = x^2 + x^3$

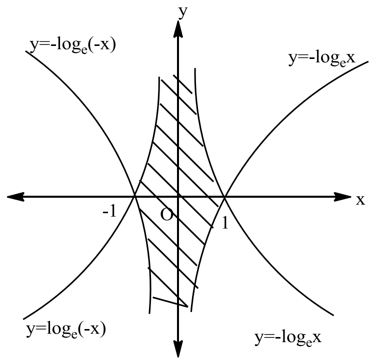
$\Longrightarrow F(x) = x + x^{\frac{3}{2}}$

$f(x) = F'(x)$ ... [From (i)]

$\Longrightarrow f(x) = 1 + \frac{3}{2}x^{\frac{1}{2}}$

$\Longrightarrow f(16) = 1 + \frac{3}{2}(16)^{\frac{1}{2}} = 7$

82 **(4)**

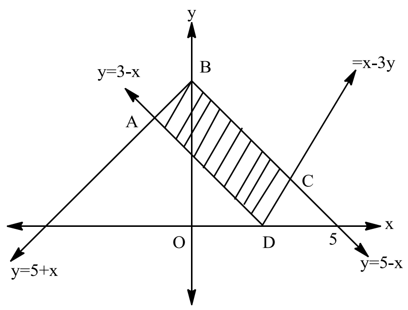


$\Longrightarrow$ Required area $= 4\int_0^1 -\log x\,dx$

$= -4[x\log x - x]_0^1$

$= 4$

83 **(64)**



$A \equiv (-1,4)$

$B \equiv (0,5)$

$C \equiv (4,1)$

$D \equiv (3,0)$

Area enclosed by curves C_1 and C_2 is area of □AB

.

Area of □ABCD $= I_1 + I_2 + I_3$

where $I_1 = \int_{-1}^{0} \big((5+x)-(3-x)\big)dx$

$= \int_{-1}^{0} (2+2x)dx = [2x+y^2]_{-1}^{0}$

$= 2(0+1) - 1 = 1$

$I_2 = \int_{0}^{3} \big((5-x)-(3-x)\big)dx$

$= \int_{0}^{3} 2\,dx = 6$

$I_3 = \int_{3}^{4} \big((5-x)-(3-x)\big)dx$

$= \int_{3}^{4} (8-2x)dx = [8x-x^2]_3^4$

$= 8(4-3) - (16-9) = 1$

$\Rightarrow$ Area of $\square$ABCD $= 1 + 6 + 1 = 8$

$\Rightarrow$ Side length of hte square $= 8$

$\Rightarrow$ Area of the square $= 64$

84 **(30)**

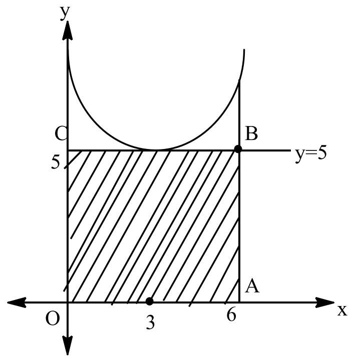
$gof(x) = g\big(f(x)\big)$

$= g(x-3)$

$= (x-3)^2 + 5$

$\Rightarrow h(x) = (x-3)^2 + 5 \geq 5$

$\Rightarrow \min\big(h(x)\big) = 5$



$\Rightarrow$ Required area = area of $\square$OABC

$= 6 \times 5 = 30$

Remarks:

Required area $= \int_{0}^{6} 5\,dx = 5(6-0) = 30$

85 **(3)**

$y = \frac{a^2-ax}{1+a^4}$ (1)

$y = \frac{x^2+2ax+3a^2}{1+a^4}$ (2)

Point of intersection of (1) and (2)

$\frac{a^2-ax}{1+a^4} = \frac{x^2+2ax+3a^2}{1+a^4}$

$(x+a)(x+2a) = 0$

$x = -a, -2a$

Req. area $= \int_{-2a}^{-a} \left[\left(\frac{a^2-ax}{1+a^4}\right) - \left(\frac{x^2+2ax+3a^2}{1+a^4}\right)\right]$

$\therefore f(a) = \frac{a^3}{6(1+a^4)}$

$f(a)$ is max is

Then $f'(a) = 0$

$3 + 3a^4 - 4a^4 = 0$

$a^4 = 3$